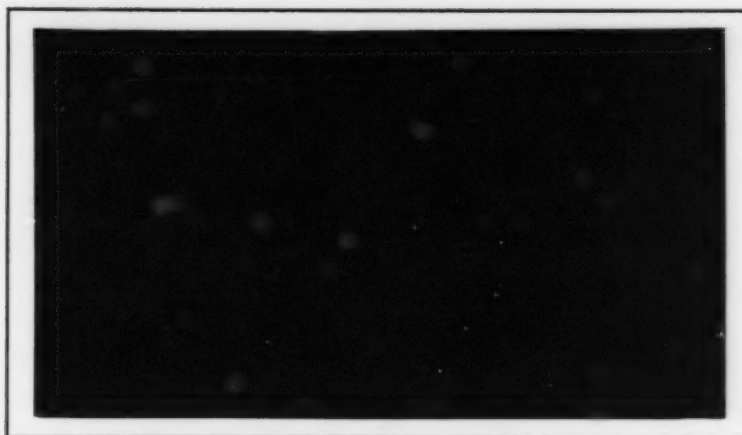




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**On Improving the Accuracy of
Line Extraction in Hough Space**

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On Improving the Accuracy of Line Extraction in Hough Space

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On Improving the Accuracy of Line Extraction in Hough Space

Summary

This paper presents the Hierarchical Peak Compaction Hough Transform (HPCHT) for line extraction. The method is robust and accurate in spite of errors in the image data. In the standard methods, when ρ is calculated using the θ values in the transform mapping from the x - y to the θ - ρ coordinates, the precision of ρ is directly affected by the errors in θ . As a result, the edge points form a vague cluster instead of voting precisely to a single (θ, ρ) cell. The ρ -scattering is further aggravated when the edge points in the image are distant from the foot of normal of the underlying edge line. This effect becomes prominent in large images where the edge points could be quite far away from their foot of normal.

In this paper, we propose the HPCHT in which an original image is first subdivided recursively into small subimages so that the ρ -scattering subsides to insignificance. Since vote clusters are compact and separable, peak detection becomes as simple as searching for connected components. The average of the θ and ρ values within a cluster gives a good statistical mean value of the peak which represents the short segment in the local subimage. Next, the short segments are merged recursively to form long lines. At each level, four child subimages are merged to form the current one. The Hough Transform is performed to collect votes from the line segments in the child subimages. The votes from each segment carries a weight proportional to the number of edge points from its local support. The scale of quantization of the Hough space is adaptive to the accuracy of the merging segments so that the θ - ρ grids will neither be too coarse such that separate clusters are overlapping nor be too fine such that a single cluster is split

unnecessarily. Henceforth, the vote clusters can always be treated as connected components and the peak detection is straightforward.

When segments merge at consecutive levels, the θ and p values become more accurate through the averaging process. As a result, the hierarchical process yields more and more accurate lines at higher levels. In our experiments, the HPCHT was tested with real image data. Lines were extracted from a 256×256 cube image and a 512×512 image of an office scene at a precision of $\pm 1^\circ$ and ± 3 pixels and these results were compared to those using the standard HT methods.

The hierarchical approach to Hough Transform has also been explored by other researchers. However, little has been done to deal with the problems of inaccurate edge data. We show that errors in the gradient direction affect the extent of vote scattering, which directly determines the sampling rate and also the peak detection strategy for the vote clusters in the Hough space. The HPCHT adapts to the error scale, thus enabling compact vote clustering and therefore reliable peak detection. Moreover, the hierarchical structure is suited for parallel implementations on pyramid computers and other similar architectures. The idea can be extended to detection of non-linear patterns like circles, or detection of patterns of arbitrary shape using the Generalized Hough Transform.

On Improving the Accuracy of Line Extraction in Hough Space

Abstract

This paper presents a robust and accurate line extraction technique called the Hierarchical Peak Compaction Hough Transform (HPCHT). Vote scattering in the parameter space is a problem when using the Hough Transform for line extraction. This paper investigates the effects of image size and edge data errors on the severity of vote scattering. The HPCHT uses a hierarchical approach based on the image pyramid. It adapts to the error scale at each level and calculates the best achievable accuracy for line estimation. The algorithm can be extended to detection of arbitrary patterns and is suited for fast parallel implementations on pyramid computers.

keywords: Hough Transform, adaptive quantization, line extraction, peak compaction, pyramid, vote scattering.

1. Introduction

The Hough transform (HT) has been recognized as a powerful technique for image shape analysis since it was first introduced as a method of detecting patterns of points in image data [6, 12]. Line extraction in the HT is an algorithm to evaluate the invariant parameters of a set of collinear image points. The essence of the method lies in the transform from an image to a parameter space in such a way that spatially extended image patterns are mapped to spatially compact peaks in the parameter space. The difficult task of global feature detection is thereby transformed into a relatively easier local peak detection problem.

Consider a set of image points $\{(x_i, y_i) \mid i = 1, \dots, n\}$ on a line L in the image space satisfying the following equation:

$$L: \rho = x_i \cos \theta + y_i \sin \theta, \quad i = 1, \dots, n \quad (1)$$

Each point (x_i, y_i) is mapped to a sinusoidal curve in the parameter θ - ρ space (also called the Hough space). As the image points are collinear on the line L , their corresponding sinusoidal curves will all theoretically intersect at a particular (θ, ρ) which represents a line containing all the image points corresponding to L . The task then becomes the detection of a high count of votes at (θ, ρ) .

The performance of the HT can be improved using the gradient information, which can be extracted during edge detection. With the gradient direction θ known a priori, the transform from (x, y) to (θ, ρ) space is restricted to a one-to-one mapping. Using the gradient direction is, however, not only a consideration of efficiency but also an issue of accuracy. In the work of Cohen and Toussaint [3], false votes due to the accidental alignment of non-collinear image points were studied quantitatively. O'Gorman and Clowes [5] suggested using the gradient direction to eliminate the irrelevant votes which do not contribute to the line of interest. The idea was extended by Van Veen and Groen in their method of gradient weighted HT [13].

Lines are detected when consensus among the votes occurs. Normally, votes scatter in clusters in the Hough space. Consensus is realized when the votes form a peak in a cluster. However, detecting peaks in the Hough space is difficult since the votes rely on the gradient direction which is subject to errors from various sources. Essentially, errors in the gradient direction θ propagate to ρ , leading to votes to an erroneous (θ, ρ) accumulator cell in the parameter space. As the constituent votes are dislocated, the shape and location of the cluster peak are corrupted accordingly.

In this paper, we investigate the problem of vote scattering due to inaccurate estimation of the edge gradient direction θ . The relationship of increased scattering and the image size will be delineated. We propose a solution, the Hierarchical Peak Compaction Hough Transform (HPCHT), which restricts the scattering of the votes by first partitioning the image into smaller subimages. Restricting the size of the subimages controls the scattering to an insignificant scale and thus a reliable HT is achieved. The method of recursive image partitioning leads to a hierarchical merging of subimages. Short segments extracted in small subimages vote for long lines in larger parent images. Accuracy is gained when multiple segments merge together. Thus, the lines extracted are very accurate when they get to the top level of the merging hierarchy. Continuity of line segments extending over several subimages are captured by using overlapping subimage windows. This hierarchical approach fits well with the popular pyramidal approach in multi-resolution image analysis.

The next section investigates the factors accounting for vote scattering in the Hough space. In Section 3, the Hierarchical Peak Compaction Hough Transform is presented. The issues of vote scattering and line extraction accuracy are dealt with in Section 3.1. Section 3.2 explains the adaptive quantization and how the peak detection becomes simplified to connected component calculation under low scattering of votes within clusters. Experimental results are presented in Section 4. Section 5 then concludes the paper.

2. Problem with Inaccurate Gradient Direction

When using the gradient direction θ in the HT, the voting process is guided by the θ estimate of the edge points. Any error in θ will affect the computation for ρ . As a result, an incorrect (θ, ρ) value is obtained and a vote will be cast to a wrong (θ, ρ) cell.

Gradient information is derived directly from the edge detection process. Canny's analysis [2] has shown the inverse relationship between the detection and localization performance of step edge detectors. Trade off in the accuracy of one or the other is inevitable. Another source of error is due to the discretization of image space. Edge calculation based on finite samples from the local neighbourhood inevitably yields low accuracy. Moreover, the interaction of responses from nearby edges also produces noise which affects the accuracy of the edge calculation. For example, Figure 2-1 presents an image (256×256) of a single edge. The edge line in Figure 2-1a passes through the origin of the image space (the

center of the image) and has a direction $\theta = 45^\circ$. A normally distributed error $\Delta\theta$ (zero mean, $\sigma_{\Delta\theta} = 5^\circ$) has been added to the edge direction to illustrate how the error in θ results in vote scattering in the θ - ρ space (Figure 2-1b). Instead of voting consistently to a sharp peak at $(45^\circ, 0)$, each individual edge point calculates ρ offset from 0 due to the error in θ . Some of the votes are accumulated to (θ, ρ) cells far from $(45^\circ, 0)$ resulting in a cluster scattering over a wide range. Figure 2-1b shows a spread of almost 30° along the θ -axis and a quarter of the image size along the ρ -axis.

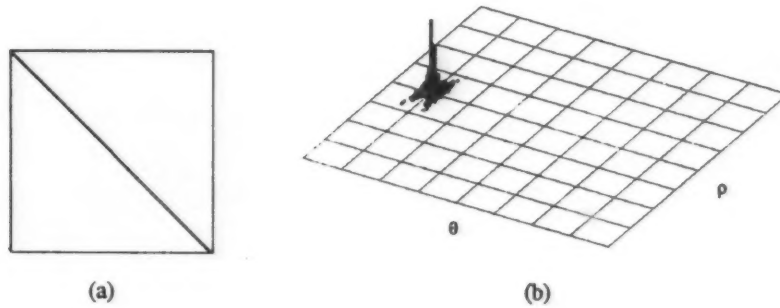


Figure 2-1:

Vote scattering in the θ - ρ space in the presence of error in the θ value. (a) An image (256×256) of a simple edge, $\bar{\theta} = 45^\circ$, $\sigma_{\Delta\theta} = 5^\circ$. (b) Vote distribution in the θ - ρ space.

In real images where many edges are present, clusters as wide and fuzzy as this would likely run into each other, making interpretation for cluster peaks very difficult. An example is taken from a 256×256 image of two rubic cubes. The edge map is shown in Figure 2-2a. The gradient directions of the edges are calculated with a 3×3 Sobel-like method [11]. It is observed that the error in the gradient direction θ has a distribution whose standard deviation is estimated to be approximately 5° . Figure 2-2b shows the votes in the corresponding θ - ρ space. Note that most of the vote clusters are not sharply defined, thus rendering subsequent peak detection unreliable.

The spread of votes in a cluster can be measured by the error of estimation for the ρ values ($\Delta\rho$). Eq. 1 indicates that ρ changes in accordance with θ , x and y . For small $\Delta\theta$, Δx and Δy ,

$$\begin{aligned} \Delta\rho &\approx \frac{\partial\rho}{\partial\theta} \Delta\theta + \frac{\partial\rho}{\partial x} \Delta x + \frac{\partial\rho}{\partial y} \Delta y \\ &= (-x \sin \theta + y \cos \theta) \Delta\theta + \cos \theta \Delta x + \sin \theta \Delta y \end{aligned} \quad (2)$$

Expressing $x = r \cos \phi$ and $y = r \sin \phi$, where ϕ is the radial angle and r is the radial distance of the point

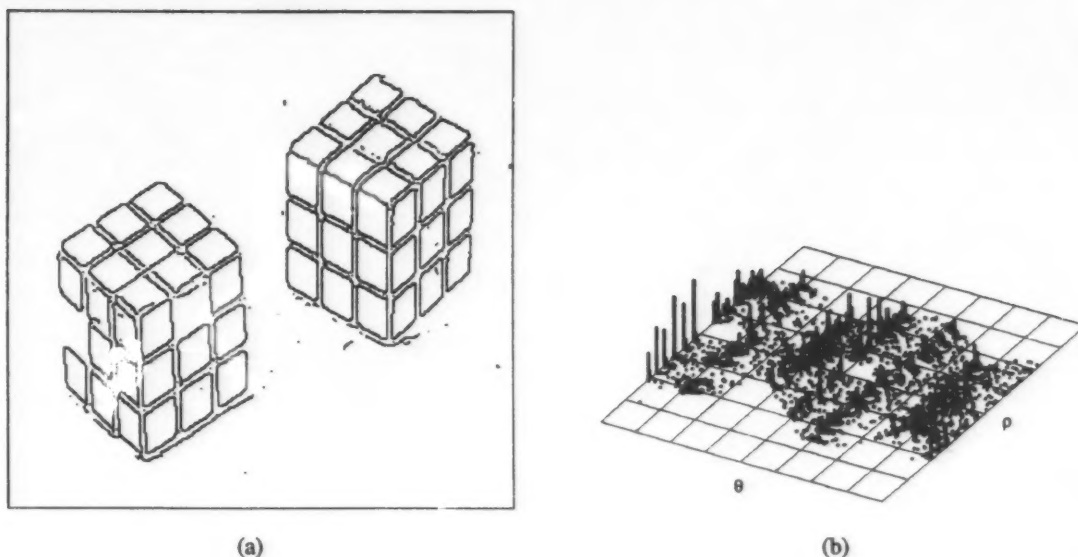


Figure 2-2:

Hough Transform on an edge image (256×256) of rubic cubes. (a) The edge map. (b) Vote distribution in the θ - ρ space. Interpretation of cluster peaks is difficult.

$P(x, y)$ from the origin (see Figure 2-3), Eq. 2 can be written as

$$\Delta\rho \approx r \sin(\phi - \theta) \Delta\theta + \cos\theta \Delta x + \sin\theta \Delta y \quad (3)$$

The terms Δx and Δy represent the error of edge localization. Usually, edge location can be estimated to pixel precision, thus minimizing or eliminating the second and third error terms in Eq. 3. The factor $r \sin(\phi - \theta)$ in Eq. 3 measures the distance of the point $P(x, y)$ from the foot of normal P_0 . It is expressed as d in Figure 2-3. Thus, we have

$$\Delta\rho \approx d \Delta\theta \quad (4)$$

Take the center of the image as the origin of the image space. In the extreme case for d , P is at the corner of the image and P_0 is at the center. Thus, the distance d could be as large as $\frac{\sqrt{2}}{2}N$ where $N \times N$ is the size of the image. Substituting $\frac{\sqrt{2}}{2}N$ for d in Eq. 4, we have

$$\Delta\rho \leq \frac{\sqrt{2}}{2}N \Delta\theta \quad (5)$$

Eq. 4 illustrates that the spread of vote clusters increases with $\Delta\theta$. Moreover, the factor d in Eq. 4

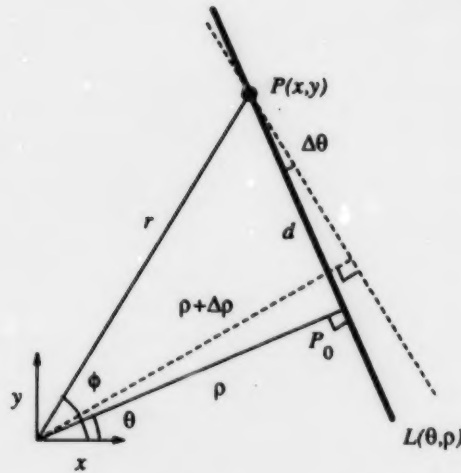


Figure 2-3:

The geometry of a line $L(\theta, \rho)$ through a point $P(x, y)$ in a 2D x - y plane. P_0 is the foot of normal of L . ϕ is the radial angle and r is the radial distance of P from the origin. d is the distance of P from P_0 . $\Delta\theta$ measures the deviation of the θ estimate at P , and $\Delta\rho$ is the error when ρ of L is computed using the inaccurate θ value at P .

amplifies the scattering of votes when the edge points are located away from the foot of normal of the underlying line. As indicated in the Eq. 5, the factor can be as large as $\frac{\sqrt{2}}{2}N$. For an image of size 256×256 , if the θ values vary with a standard deviation $\sigma_{\Delta\theta}$ of 5° , $\Delta\theta$ would be considered lying in a range of $\pm 3\sigma_{\Delta\theta}$, i.e. $\pm 15^\circ$. Then, $\Delta\rho$ could be as large as ± 48 pixels. That means the vote cluster in the θ - ρ space is likely to scatter over a range of approximately 96 pixels in the ρ direction, which was illustrated in Figure 2-1. In these cases, overlapping of nearby vote clusters will be severe, and the peak detection becomes unreliable.

The effect of d is illustrated in Figure 2-4. Figure 2-4a shows a subimage extracted from Figure 2-1a at the center of the image, and Figure 2-4b is the vote distribution in the corresponding θ - ρ space. When a subimage is extracted near the border of the image, as illustrated in Figure 2-4c, the vote cluster spreads out wider in the ρ direction (see Figure 2-4d).

Statistically, $\Delta\theta$ can be modeled approximately as a random variable with zero-mean and a standard deviation of $\sigma_{\Delta\theta}$. For normal distribution, the probability of having a $\Delta\theta \geq \sigma_{\Delta\theta}$ is about 32%.

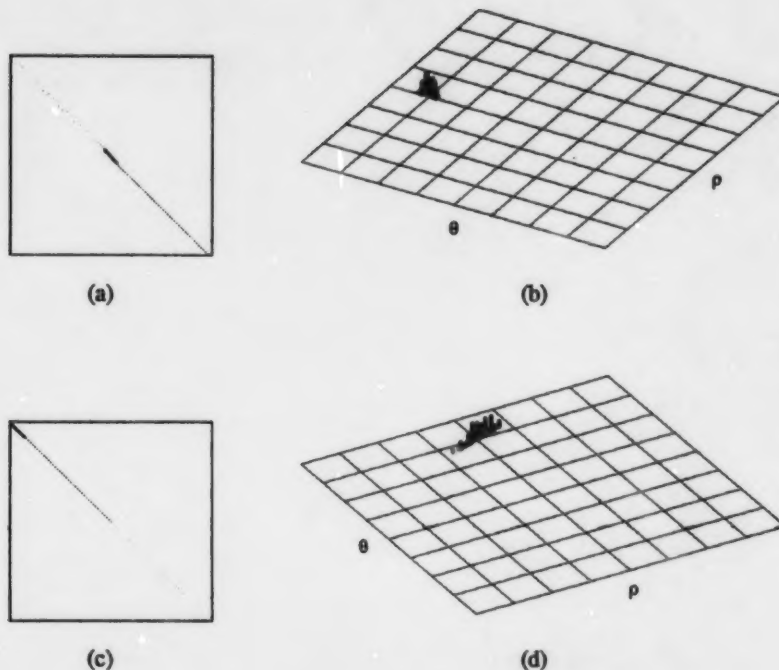


Figure 2-4:

Effects of d . (a) Edge points shown as dark pixels are extracted near the center of Figure 2-1a. (b) Vote distribution in the θ - ρ space. The votes form a relatively tight cluster. (c) Edge points near the border of Figure 2-1a. (d) Vote distribution in the θ - ρ space. The votes are more scattered when compared with (b).

Over 99% of the samples fall into the range of $\pm 3\sigma_{\Delta\theta}$ [†]. In fact, the vote scattering shown in Figure 2-2b does support the estimation for normal distribution in the $\Delta\theta$ values. Therefore, for a random distribution of $\Delta\theta$, votes will scatter along the θ -axis and a bound of $B_{\Delta\theta} = 3\sigma_{\Delta\theta}$ will cover practically all the votes in a tight cluster.

$$B_{\Delta\theta} = 3\sigma_{\Delta\theta} \quad (6)$$

[†]For a more general case, a very conservative estimation is given by the Chebyshev's Inequality which describes for any distribution that the probability of having $|\Delta\theta|$ exceeding an upper bound $B_{\Delta\theta}$ is

$$P(|\Delta\theta| \geq B_{\Delta\theta}) \leq \frac{\sigma_{\Delta\theta}^2}{B_{\Delta\theta}^2}$$

That means at least $1 - \frac{\sigma_{\Delta\theta}^2}{B_{\Delta\theta}^2}$ of the samples are contained within a range of $\pm B_{\Delta\theta}$. A bound of $B_{\Delta\theta} = 3\sigma_{\Delta\theta}$ thus always has an overwhelming probability of covering the samples.

Since a range of $\pm B_{\Delta\theta}$ covers practically all votes in θ , substituting $B_{\Delta\theta}$ for the $\Delta\theta$ in Eq. 5 yields a bound on $\Delta\rho$ which will cover most votes in ρ . Let $B_{\Delta\rho}$ be the bound on $\Delta\rho$.

$$B_{\Delta\rho} = \frac{\sqrt{2}}{2} N B_{\Delta\theta} = \frac{3\sqrt{2}}{2} N \sigma_{\Delta\theta} \quad (7)$$

This forms a rectangular region bounded by $\pm B_{\Delta\theta}$ and $\pm B_{\Delta\rho}$ centered at (θ, ρ) which has a very high probability of containing all the votes in a cluster. Therefore, the scattering effect in the θ - ρ space can be described statistically as follows: for a random distribution of $\Delta\theta$, almost all of the votes in a cluster would be scattering in a region bounded by $\pm B_{\Delta\theta}$ and $\pm B_{\Delta\rho}$ as calculated in Eq. 6 and 7.

In Figure 2-1a, $\Delta\theta$ has a normal distribution with $\sigma_{\Delta\theta} = 5^\circ$ and $N = 256$. With $B_{\Delta\theta}$ fixed at $3\sigma_{\Delta\theta} = 15^\circ$, $B_{\Delta\rho}$ is 48 pixels from Eq. 7. Therefore, the votes are expected to scatter over a range of $\pm 15^\circ$ and ± 48 pixels, which is observable in Figure 2-1b. Vote scattering in the real image as shown in Figure 2-2, where $\sigma_{\Delta\theta} = 5^\circ$ and $N = 256$, is also within the expectation of the above analysis. The vote clusters in Figure 2-2b seem to be less spread-out. The reason is because the edge point distribution in the image space is less drastic in Figure 2-2a. None of the edge points are very far from their feet of normal of the underlying fitting lines. Figure 2-4 presents the two extreme scenarios of vote scattering extracted from Figure 2-2. Note that d (which is used in Eq. 4 to measure the distance from an edge point to the foot of normal of the underlying fitting line) varies from 0 in Figure 2-4a to the full diagonal length (from the center to the corner of the image) in Figure 2-4c, whereas d in Figure 2-2a measures to less than one half of the diagonal length. It is observed that the ρ -scattering shown in Figure 2-2b lies in between those in Figure 2-4c and 2-4d. As expected, it reaches to less than half the magnitude of that in Figure 2-4d.

3. The Hierarchical Peak Compaction Hough Transform (HPCHT)

As we have shown in the previous section, the quality of the clusters in the Hough space and hence the effectiveness of peak detection is largely affected by the error in θ estimation. When N is 256 and $\sigma_{\Delta\theta}$ is 5° , $B_{\Delta\rho}$ reaches 48 pixels which is unacceptable. However, if the image were subdivided to a smaller size, say $N = 32$, with the same $\sigma_{\Delta\theta} = 5^\circ$, $B_{\Delta\rho}$ would be reduced to a small scale, 6 pixels in this case. In general, when the extent of the ρ -scattering is restricted to be smaller than the inter-cluster separation, the votes will be controlled within small and separate clusters. Therefore, the ambiguity

problem of peak detection as depicted in Figure 2-2b is alleviated since the scattering is insignificant.

To solve the scattering problem, we propose a Hierarchical Peak Compaction Hough Transform procedure (HPCHT) which has two hierarchical processing components. In the first hierarchical step, the original image is split recursively into small subimages. Vote distribution is controlled within small tight clusters. Therefore, peak detection can be accomplished by treating the vote clusters as connected components. Averaging the θ and ρ values of the edge points in the cluster yields the centroid of the peak, which is used to represent the underlying line segment. Therefore, at the bottom level of the image subdivision hierarchy, the HT can be applied effectively within the local window of each subimage and small line segments are extracted unambiguously.

In the second hierarchical step, ascending from the level next to the bottom of the hierarchy, the recursive merge procedure forms longer lines progressively using the short segments reported from below. The segment merging process also follows a HT scheme. At each level, the HT is performed to collect votes from short segments. The vote from each segment carries a weight proportional to the number of edge points from its local support. Peaks in the θ - ρ space indicate the most probable lines at the current level. Again, peak detection is done by finding the connected components and averaging the θ and ρ values for the centroid of the peak. When the top level is reached, integrated long lines are obtained.

As a long line segment takes the average of short ones, the result would be more accurate in terms of θ and ρ evaluation. When n θ values are averaged to yield the line orientation estimate, the statistical error in the result is $\sqrt{n}$ times smaller than that in each of the individual θ values. Since ρ is derived from θ , $\Delta\rho$ is also improved. Instead of growing linearly with image size (as described in Eq. 7), $\Delta\rho$ can remain bounded within a few pixels. Such an accuracy improvement enables a progressively finer quantization of the θ - ρ space. At each level, the procedure calculates the expected scattering, $B_{\Delta\theta}$ and $B_{\Delta\rho}$. These estimates are then used to fine tune the Hough space quantization. Vote peaks for long lines at the higher levels are thus located more accurately. The accuracy improvements for each cycle of the recursive merging will be explained in the next section.

The actual image subdivision uses an overlapped pyramid where overlapping among sibling subimages are possible (see Figure 3-1). Our image subdivision pyramid differs slightly from Burt's pyramid [1] in that the image is partitioned from top down into four overlapped subimages with a size ratio of $\frac{5}{8} \times \frac{5}{8}$. The overlap is to capture the continuity of the lines extending over the boundary between

adjacent subimages. The common sub-segment in the overlapping region helps ensuring the agreement on the underlying extended line between the two segments when they merge from adjacent subimages.

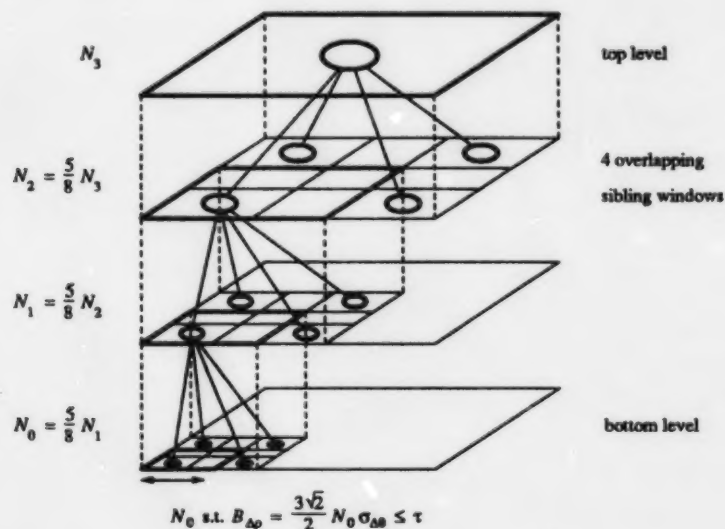


Figure 3-1:

The overlapped pyramid of image subdivision. The image at the current level is subdivided into four overlapped subimages at the next lower level. At the bottom level, the image is divided into size $N_0 \times N_0$ which is sufficiently small to overcome the ρ -scattering effect (i.e. $B_{\Delta p}$ observed in the $N_0 \times N_0$ window does not exceed the ρ -scattering tolerance, τ , allowed by the inter-cluster separation). The sibling subimages are overlapped to capture the continuity of the lines extending over the boundaries of subimages. In our implementation, the size ratio of image subdivision is $\frac{5}{8}$ in both dimensions. The factor of overlap is thus $\frac{1}{4}$.

In summary, the HPCHT procedure is as follows:

1. *Scattering Test* -- Calculate the scattering, $B_{\Delta\theta}$ and $B_{\Delta p}$, of the current image with the current size N and $\sigma_{\Delta\theta}$ using Eq. 6 and 7.
2. *Recursive Subdivision* -- If $B_{\Delta p}$ is larger than the tolerance τ allowed by the inter-cluster tolerance, say ± 6 pixel, split the image into 4 sibling subimages in the pyramid as depicted in Figure 3-1. Repeat Step (1) and Step (2) recursively until $B_{\Delta p} \leq \tau$.
3. *Hough Transform* -- Apply the Hough Transform to the subimages. Map each edge point to the Hough space according to the current θ estimate. The Hough space is quantized using the current $B_{\Delta\theta}$ and $B_{\Delta p}$ as the grid size.
4. *Peak Detection* -- Collect the votes for a peak as a connected component in the Hough space. The centroid of the peak gives the (θ, ρ) value of the underlying line. The contributing edge points are now represented by the line as a group sharing the same θ and

ρ values with the errors improved through the averaging process.

5. *Recursive Merging* -- Go up one level to the parent subimage. Calculate the new $B_{\Delta\theta}$ and $B_{\Delta\rho}$ (Eq. 12 through 15 in Section 3.2.1 will give the simplified formulae for the calculation). Perform Step (3) through Step (5) recursively to extract the lines underlying the segments until it reaches the top of the hierarchy.

6. Report the lines extracted.

3.1. Accuracy of the Line Extraction

This section discusses the accuracy of the HPCHT. The algorithm starts on small subimages where $\Delta\rho$ can be kept within the tolerance allowed by the inter-cluster separation. When line segments merge, $\Delta\theta$ is improved and which, in turn, will reassess the error bound on the ρ estimate. As a result, $\Delta\rho$ is kept low throughout, while $\Delta\theta$ improves as longer line segments are formed.

3.1.1. Precision of the θ Estimates from the HPCHT

In the hierarchy of recursive line extraction, short segments from a lower level vote for longer ones at the next higher level. Since the original edge data at the bottom level is not perfect, short line segments derived from it might not be accurate. However, the error in the θ estimates from the edge detector output does not propagate all the way up the line extraction hierarchy. The θ estimates are improved as line segments are hierarchically merged.

When a line segment is detected, the votes are collected and a peak stands out in the θ - ρ space. The position of the peak is estimated by simple averaging on the (θ, ρ) values of all the contributors. The result represents a better estimate of the (θ, ρ) value of the underlying line. To illustrate this, let $\Delta\theta_1, \dots, \Delta\theta_n$ be the $\Delta\theta$'s of n collinear image points. Suppose they are independent random variables with the same probability distribution. Let the standard deviation of the distribution be σ . The average of the n $\Delta\theta$'s also has a random distribution but with an improved standard deviation. The factor of improvement is $\sqrt{n}$. That means if σ is the standard deviation and the θ estimate of each individual point mostly lies within $\pm\sigma$ from the true direction θ^* , then the resulting average is likely to be found within $\pm\frac{\sigma}{\sqrt{n}}$ about θ^* . As longer lines are formed higher up the hierarchy, the statistical error in the resulting θ

estimates would be reduced progressively. For the lines with length of n edge points, the accuracy of the edge orientation achieved at level i , $\sigma_{\Delta\theta_i}$, will be better than the initial estimate, $\sigma_{\Delta\theta_E}$, obtained from the edge detector by a factor of $\sqrt{n}$. Suppose the length of line segments runs in the order of image size N_i ,

$$\sigma_{\Delta\theta_i} = \frac{\sigma_{\Delta\theta_E}}{\sqrt{N_i}} \quad (8)$$

Figure 3-2 shows the improvement of θ estimation achieved in the hierarchical process of line extraction. The edge pixels start with a $\sigma_{\Delta\theta_E} = 5^\circ$. The θ error drops as long segments are formed from bottom up in the hierarchy.

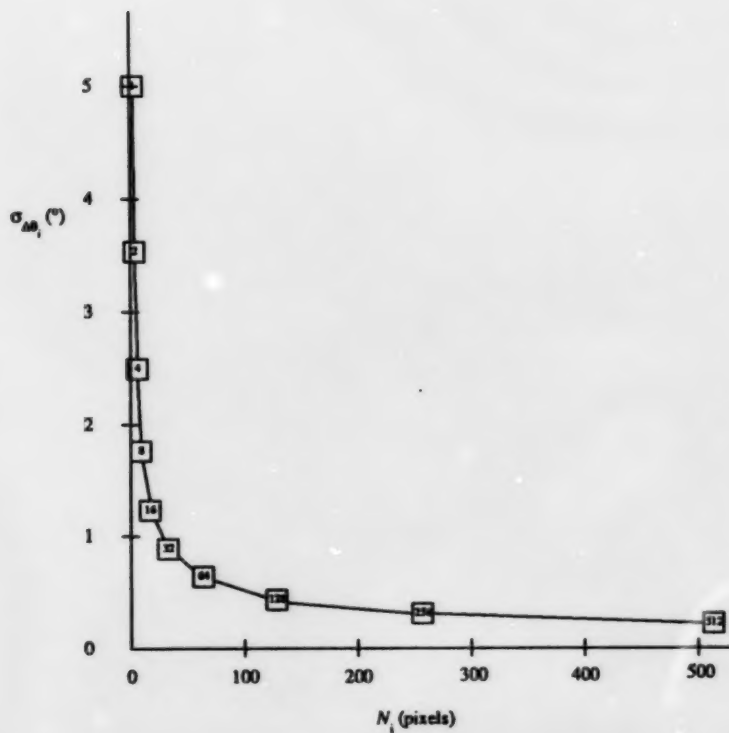


Figure 3-2:

The trend of decreasing $\sigma_{\Delta\theta_i}$ with subimage size N_i increasing up the hierarchy. The initial $\sigma_{\Delta\theta_E}$ is 5° .

3.1.2. A Low ρ -Scattering

Vote clusters must be disconnected from each other in order for them to be distinguishable. As shown in Figure 2-4, vote clusters are more centralized and peak detection based on connected components is easier for the edge points located in small windows at the center of the image. In fact, the argument stands quite distinctively from the mathematical behaviour of ρ -scattering. Eq. 5 gives an upper bound of the ρ -scattering. Restricting the image size will control the ρ -scattering.

With the bound given in Eq. 7, we partition the image recursively until $\Delta\rho$ is contained within the ρ -scattering tolerance τ . For the cube image in Figure 2-2a, we chose a range of ± 6 pixels for τ .

$$\begin{aligned} B_{\Delta\rho} &= \frac{3\sqrt{2}}{2} N \sigma_{\Delta\theta} \leq \tau = 6 \\ N &\leq \frac{\sqrt{2}\tau}{3\sigma_{\Delta\theta}} = \frac{2\sqrt{2}}{\sigma_{\Delta\theta}} \end{aligned} \quad (9)$$

An error of $\sigma_{\Delta\theta} \approx 5^\circ$ in the θ values is observed in Figure 2-2a. By Eq. 9, the choice for N will be 31.

$$N \leq \frac{2\sqrt{2}}{\frac{5}{180}\pi} = 31$$

This means that an image when subdivided into size no larger than 31×31 pixels will always keep the vote clusters centralized within ± 6 pixels along the ρ -axis. At that scale of scattering, the vote clusters are compact and well separated. Cluster isolation and peak localization become straightforward. A simple strategy for peak detection will be discussed in the next section.

The ρ -scattering tolerance τ has a direct relationship with the inter-cluster separation. For images in which the edges are dense, vote clusters will not be far from each other. A less generous tolerance on ρ -scattering is used to avoid collisions between clusters. In some cases, τ might be set at ± 3 pixels or even smaller. Thus, the small subimages to start with would be restricted to 15×15 or smaller. The result is a deeper hierarchy of image subdivision and segment merging, and henceforth, a longer execution time of the algorithm. In fact, the spatial separation between parallel lines[†] in the original image corresponds to the inter-cluster separation in the Hough parameter space, and hence represents a

[†]Note that parallel lines are lines with the same θ value. Opposite edges of a rectangular image pattern are anti-parallel, i.e. their θ values differ from each other by 180° .

good criterion for the choice of the ρ -scattering tolerance. Parallel lines in an image share the same θ value but have different ρ values. Their vote clusters thus will line up in the θ - ρ space on the same ρ axis. The ρ -scattering, as long as it does not exceed the allowance by the ρ separation of the vote clusters, does not obscure the cluster isolation and peak localization in the subsequent stages of the Hough process. As an example, the cube image in Figure 2-2 does not have dense edges. The parallel lines are no less than 12 pixels apart. In our tests, the HPCHT performed satisfactorily with a relaxed tolerance of $\tau = 6$ pixels and a hierarchy of 5 levels.

Segments are extracted in small subimages at the bottom level of the image subdivision hierarchy. The segments are then merged recursively from bottom up. At level i , four child images of size $N_{i-1} \times N_{i-1}$ are merged to the current image of $N_i \times N_i$. During the process, the θ and ρ values of the child segments are recalculated with reference to the coordinate system of the current image space. The calculation essentially involves a translation of the origin of the child space to the current space.

$$\rho_i = \rho_{i-1} + x_{o_{i-1}} \cos \theta_{i-1} + y_{o_{i-1}} \sin \theta_{i-1} \quad (10)$$

where θ_{i-1} , ρ_{i-1} , ρ_i are the θ and ρ values calculated over the two levels respectively; $(x_{o_{i-1}}, y_{o_{i-1}})$ is the position of the child space's origin measured in the current space's coordinate system (see Figure 3-3).

Applying the similar treatment as in Eq. 2 through Eq. 4, we have the error in the terms $x_{o_{i-1}} \cos \theta_{i-1} + y_{o_{i-1}} \sin \theta_{i-1}$ of Eq. 10 calculated to $d \Delta \theta_{i-1}$, where $\Delta \theta_{i-1}$ is the error in the θ estimate at the child level and d is the distance of $(x_{o_{i-1}}, y_{o_{i-1}})$ from the line normal in the current space which is corresponding to the translational displacement of the foot of normal from the child space to the current one. For simplicity of the analysis, we assume the child subimage windows do not overlap. Therefore, d measures to the order of $\frac{\sqrt{2}}{2} N_{i-1}$. Thus, we have

$$\Delta \rho_i \approx \Delta \rho_{i-1} + \frac{\sqrt{2}}{2} N_{i-1} \Delta \theta_{i-1}$$

With $\Delta \theta_{i-1}$ and $\Delta \rho_{i-1}$ being considered as independent random variables, $\Delta \rho_i$ is treated as a random variable as well. Averaging the $\Delta \rho_i$'s among the child segments yields a result with a variance which is effectively half of the original values, since the length of the segment is normally assumed being doubled in the merging process.

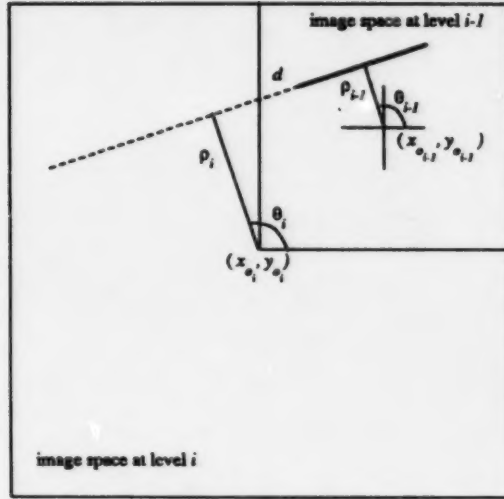


Figure 3-3:

The θ and ρ values of the child segments are recalculated with reference to the current space's coordinate system during the merging process. It essentially involves a translation of the origin of the child space to the current space. (x_{o_i}, y_{o_i}) and $(x_{o_{i-1}}, y_{o_{i-1}})$ are the origins of the image spaces in the levels i and $i-1$ respectively; θ_i and θ_{i-1} are the θ estimates; ρ_i and ρ_{i-1} are the ρ estimates and d measures the translational displacement of the foot of normal.

$$\sigma_{\Delta\rho_i}^2 = \frac{1}{2} \left(\sigma_{\Delta\rho_{i-1}}^2 + \frac{1}{2} N_{i-1}^2 \sigma_{\Delta\theta_{i-1}}^2 \right)$$

Such a recurrence relation yields

$$\sigma_{\Delta\rho_i}^2 = \frac{N_0}{N_i} \sigma_{\Delta\rho_0}^2 + \frac{1}{6} \left(1 - \left(\frac{N_0}{N_i} \right)^2 \right) N_i \sigma_{\Delta\theta_E}^2$$

Level 0 is the bottom level in the hierarchy where the image is partitioned to a size of N_0 such that the ρ -scattering, $B_{\Delta\rho_0}$, is limited to τ pixels (the calculation for N_0 is given in Eq. 9). Taking $B_{\Delta\rho_0} = 3 \sigma_{\Delta\rho_0} = \tau$,

$$\sigma_{\Delta\rho_i} = \left(\frac{N_0}{9N_i} \tau^2 + \frac{1}{6} \left(1 - \left(\frac{N_0}{N_i} \right)^2 \right) N_i \sigma_{\Delta\theta_E}^2 \right)^{\frac{1}{2}} \quad (11)$$

Eq. 11 can be understood as composed of two components. The first component inside the square root

behaves like the precision improvement when segments merge (the argument is similar to that for the improving $\sigma_{\Delta\theta_i}$ as discussed in the previous section). It dominates at small image size N_i , causing the overall $\sigma_{\Delta p_i}$ drops initially with increasing N_i . The second component inside the square root represents a combination of precision improvement and error aggravation when short segments from small child subimages merge to a long length but in a larger parent subimage (Eq.5 explains the error aggravation effect). The result is a slow growing trend of $\sigma_{\Delta p_i}$ which becomes observable at large image size N_i . Figure 3-4 shows the trend of p -scattering $\sigma_{\Delta p_i}$ with respect to image size N_i . It defies the linear growing rate expressed in Eq. 5 for the standard HT methods. Note that for images with denser edge distribution, the p -scattering tolerance τ is less generous, requiring the image to subdivide into smaller subimages to restrict the p -scattering to a scale within the tolerance. As a result, for a more restrictive τ , N_i starts at a smaller N_0 and spans a deeper hierarchy.

In our algorithm, the p -scattering $\sigma_{\Delta p}$ caused by data error $\sigma_{\Delta\theta_E} = 5^\circ$ in a large image like 512×512 does not exceed ± 1 pixel in standard deviation at the top level of the hierarchy. Therefore, a practical bound of ± 3 pixels is sufficient to contain the votes in a tight cluster. The method can extract parallel edges and distinguish between them even when they are only a few pixels apart. For example, if the edge separation is no less than 6 pixels for an image of size 512×512 , with τ fixed at 3 pixels, the HPCHT can compute edge line localization with subpixel accuracy. Compared with the scattering of ± 32 pixels, or ± 96 in the worst case, exhibited by the standard methods, the HPCHT actually achieves a tremendous accuracy improvement.

3.2. Peak Detection in the Low Scattering Hough Space

A line segment in the image space is evident when the votes from its edge pixels demonstrate a peak in the Hough space. Depending on the pattern of the vote distribution, different peak detection strategies have to be devised to extract the peaks correctly. For a distribution with separable clusters, a feasible method is to simply isolate individual clusters. The centroid of each cluster will give a good statistical mean value of the peak.

The question is: what are the criteria for the separability of the clusters? Normally, the Hough space is implemented as a discrete space of accumulator cells. The grid size has to be tuned carefully to achieve cluster separability on one hand, and to keep cells in individual clusters connected on the other

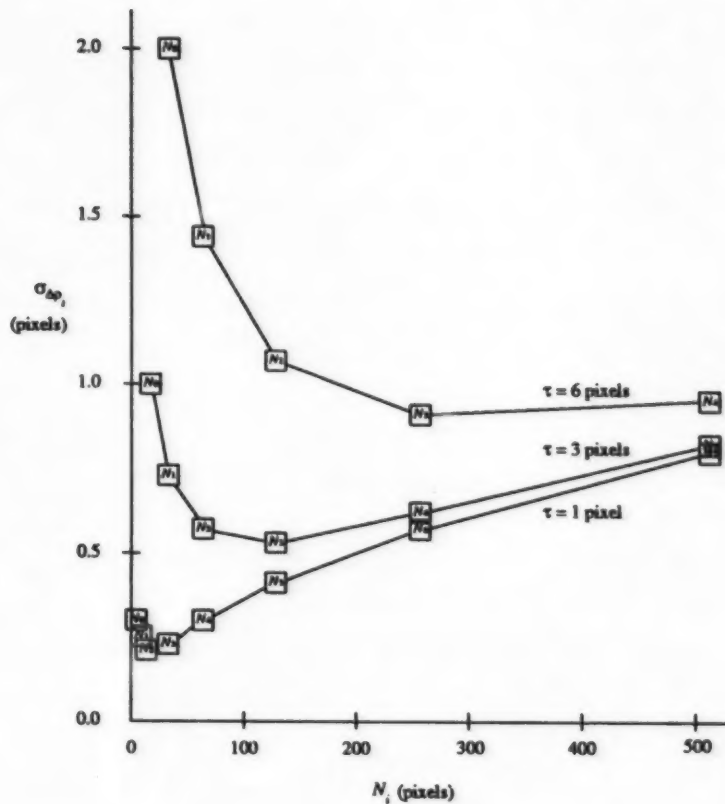


Figure 3-4:

The trend of $\sigma_{\Delta \rho_i}$ at different τ values when image size N_i increases up the subimage hierarchy. The initial $\sigma_{\Delta \theta_0}$ is 5° .

hand. Grid sizes which are too coarse will cause cluster collision; grid sizes which are too fine will split a cluster.

In the HPCHT, the quantization uses an adaptive scale that guarantees votes to the same cluster will stay within adjacent cells. Cluster isolation in such a low scattering Hough space is thus simplified to finding connected components. Moreover, with the low ρ -scattering and increasing θ -precision up the subimage hierarchy, the quantization of the θ - ρ space is refined along the θ axis progressively to achieve a HT of high resolution.

3.2.1. Quantization of the θ - ρ Space

Edge points do not vote precisely to a single (θ, ρ) due to the inaccuracy of their θ and ρ estimates. If $B_{\Delta\theta}$ and $B_{\Delta\rho}$ are the range of variation of the respective estimates, and θ^* , ρ^* are the true values, then the votes most likely fall within the region of $\theta^* \pm B_{\Delta\theta}$ and $\rho^* \pm B_{\Delta\rho}$. In our method, the θ - ρ space is quantized by using $B_{\Delta\theta}$ and $B_{\Delta\rho}$ as the grid size. All the votes belonging to the same cluster will scatter over no more than a few cells for most of the cases. This is illustrated in Figure 3-5 which shows the vote distribution of a small subimage of Figure 2-2a.

Eq. 8 and 11 calculate the vote scattering at each level of the hierarchical merge, but assuming minimum scattering which is achievable by long connected segments running the whole length or width of the subimage. Although short segments when merging at level i do not achieve the same accuracy as expected by Eq. 8 and 11, to avoid irrelevant grouping and guarantee for collinearity, the best achievable line accuracy as predicted by the equations is always assumed. The result is that the merging process always yields lines of high degree of collinearity which are either long connected edge lines or partially occluded edges lined with highly collinear segments. Extraction of lines which are short but stand-alone will not be affected, however, because a short connected segment before merging to its full length follows the accuracy calculation of Eq. 8 and 11 and stays as a single whole segment throughout the process afterwards.

According to Eq. 8 and 11, accuracy of the best achievable lines at each level is improved during the process of hierarchical merging. There is a gain in θ -precision from level to level yet the ρ -scattering always remains low. As a result, when subimages merge, collinear segments extracted from different subimages are expected to vote to a peak which is more refined than the peaks obtained at the level below. To take advantage of the precision gain, quantization of the Hough space is refined accordingly to increase the resolution in θ estimation of the line extraction. In other words, the Hough space is always quantized with respect to the $B_{\Delta\theta}$ and $B_{\Delta\rho}$ of the votes for the long segments expected at the current level.

During the voting process at level i , segments from one level below vote to the Hough space at the current level. Generally, two segments are merged and their average becomes the new θ and ρ estimates for the integrated line. The precision achieved in the newly formed line is measured with $\sigma_{\Delta\theta_i}$ and $\sigma_{\Delta\rho_i}$ which are calculated using Eq. 8 and 11. Section 3.1.1 has already explained about the $\sqrt{n}$ -fold error reduction when averaging among n edge points. Based on the same argument, when the two segments

from level $i-1$ are averaged to give the integrated line at level i , the error is improved by $\sqrt{2}$ times. This means that votes in the Hough process of level i are likely to scatter over a range of

$$B_{\Delta\theta_i} = 3 \sqrt{2} \sigma_{\Delta\theta_i}$$

$$B_{\Delta\rho_i} = 3 \sqrt{2} \sigma_{\Delta\rho_i}$$

In practice, we do not consider subpixel $B_{\Delta\rho}$ because a precision of $B_{\Delta\rho}$ at that level will be overshadowed by the edge pixel localization error anyway. Combining with Eq. 8 and 11, we have

$$B_{\Delta\theta_i} = \frac{3 \sqrt{2} \sigma_{\Delta\theta_E}}{\sqrt{N_i}} \quad (12)$$

$$B_{\Delta\rho_i} = \max \left\{ 1, \left(\frac{2 N_0}{N_i} \tau^2 + 3 \left(1 - \left(\frac{N_0}{N_i} \right)^2 \right) N_i \sigma_{\Delta\theta_E}^2 \right)^{\frac{1}{2}} \right\} \quad (13)$$

Eq. 12 and 13 calculate the scattering, $B_{\Delta\theta_i}$ and $B_{\Delta\rho_i}$, and thus the grid size of the θ - ρ space for levels of $i > 0$. In the limiting case, lines at level 0 are extracted using the standard Hough methods. As the θ values are read from the edge data and the ρ -scattering is controlled to τ pixels by subdividing the image,

$$B_{\Delta\theta_0} = 3 \sigma_{\Delta\theta_E} \quad (14)$$

$$B_{\Delta\rho_0} = \tau \quad (15)$$

3.2.2. Peak Detection by Finding Connected Components

Figure 3-6 shows the vote distribution of the subimages from the bottom three levels of the subimage hierarchy which is implemented as an overlapped pyramid. With $\sigma_{\Delta\theta_E} = 5^\circ$, the cubes image from Figure 2-2a is partitioned from the original size $N_4 = 256$ into subimages of $N_2 = 100$, $N_1 = 62$ and $N_0 = 38$ at the bottom three levels according to Figure 3-1 (Eq. 9 calculates N_0 to 32 but $N_0 = 38$ is close enough). Segments are extracted at the bottom level and merged to one level higher. The averaging process reduces the $\sigma_{\Delta\theta}$, $\sigma_{\Delta\rho}$ and thus the vote scattering. From the calculations in Eq. 12 through 15, $B_{\Delta\theta_0}$ starts at 15° and is reduced to $B_{\Delta\theta_1} = 2.7^\circ$ and then to $B_{\Delta\theta_2} = 2.1^\circ$. Likewise, $B_{\Delta\rho_0} = 6$ pixels, $B_{\Delta\rho_1} = 6$ pixels and $B_{\Delta\rho_2} = 4.5$ pixels. Note that the grid size is fixed at $B_{\Delta\theta_i}$ and $B_{\Delta\rho_i}$. Although the quantization of the Hough space is refined from level to level, the vote scattering remains well under

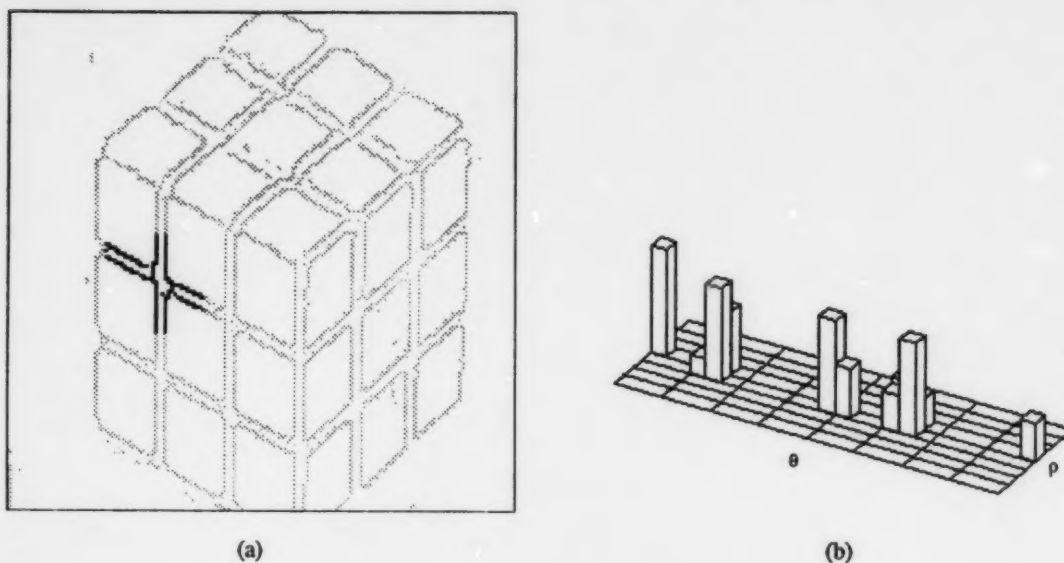


Figure 3-5:

Vote scattering can be kept to minimum by restricting the HT to small subimages and choosing the achievable $B_{\Delta\theta}$ and $B_{\Delta\rho}$ for the parameter space quantization. (a) A 32×32 subimage of Figure 2-2a. In the standard HT methods, the $B_{\Delta\theta} = 15^\circ$ and the $B_{\Delta\rho} = 6$ pixels according to Eq. 6 and 7. (b) The vote distribution with the θ - ρ space quantized at 15° and 6 pixels.

control. In Figure 3-6, the peaks are sharp and well-separated. The scattering is subsumed by the quantization of the Hough space.

Since each individual vote cluster is well in focus and spreads over no more than a few adjacent cells, the peak detection process is straightforward. It only needs to inspect the adjacent θ - ρ cells to discover the connected components. Each of these isolated connected components stands for a vote cluster, and its position can be determined by the average of all the contributing votes. The arithmetic mean value can be used to estimate the central position of a vote cluster because the clusters generally are not spatially extended.

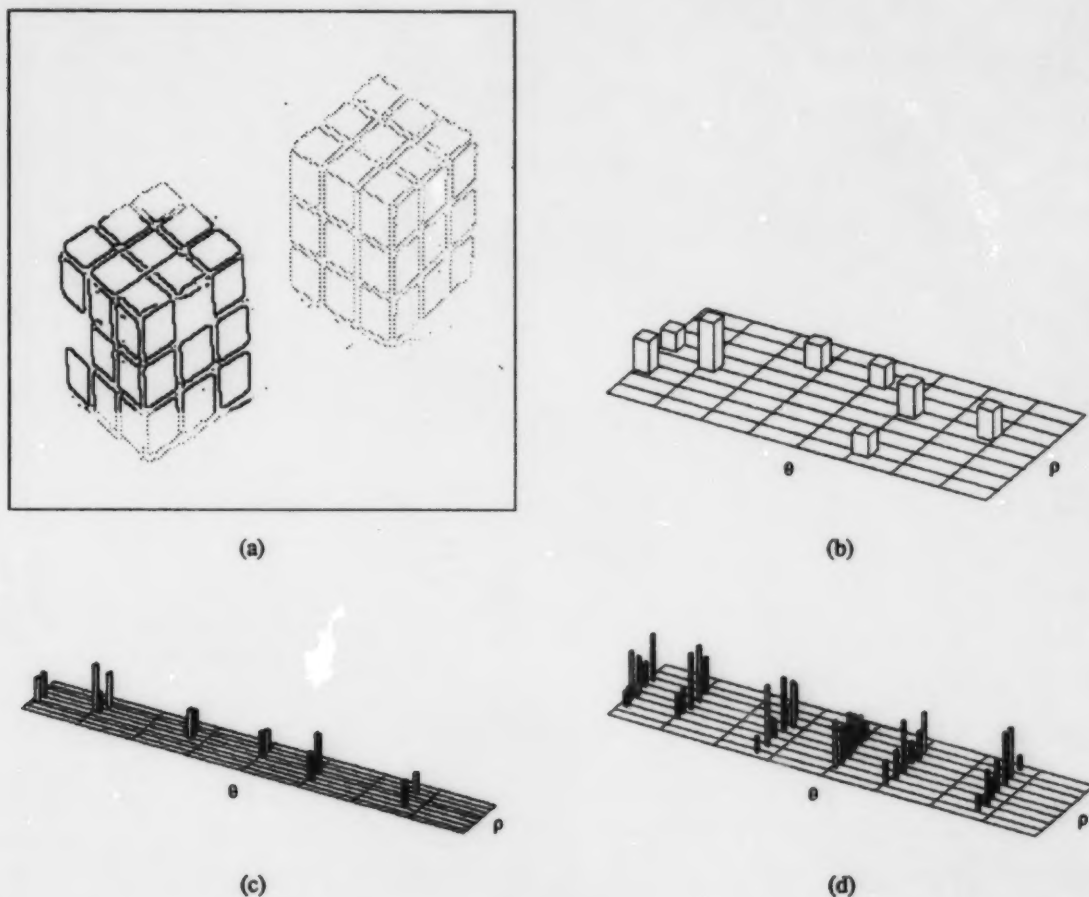


Figure 3-6:

The vote distribution of the subimages from three levels of the subimage hierarchy. The top level is a 100×100 subimage extracted from Figure 2-2a. Splitting in a pyramid as shown in Figure 3-1, the subimage is divided from $N_2 = 100$ into four overlapped subimages of $N_1 = 62$, each of which is then further split into four subimages of size $N_0 = 38$. The size ratio of splitting is $\frac{5}{8}$. (a) A 100×100 subimage of Figure 2-2a. The initial θ error has a standard deviation $\sigma_{\Delta\theta_0}$ of 5° . (b) The vote distribution of a 38×38 subimage from level 0. The θ grid size is $B_{\Delta\theta_0} = 15^\circ$, $B_{\Delta\rho_0} = 6$ pixels. (c) The vote distribution of a 62×62 subimage located at level 1 of the pyramid. The image is formed by merging four overlapping 38×38 subimages. The θ grid size is $B_{\Delta\theta_1} = 2.7^\circ$, $B_{\Delta\rho_1} = 6$ pixels. (d) The vote distribution of the subimage from Figure (a). It is from level 2 and is formed by merging four overlapping 62×62 subimages from level 1 of the pyramid. The θ grid size is $B_{\Delta\theta_2} = 2.1^\circ$, $B_{\Delta\rho_2} = 4.5$ pixels.

4. Experimental Results

The proposed HPCHT has been implemented and applied on various images of real scenes. In this section, the results from two examples will be presented.



Figure 4-1:

Two examples used in the experiments. (a) A 256×256 image of two rubic cubes. (b) An office scene. The size is 512×512 .

Figure 4-1a shows the 256×256 image of two rubic cubes. The image has long edge lines which are good for testing the line detectability of the algorithm. Note that the parallel lines in the image share the same θ but have different ρ 's. In addition, segments on one cube (like those on the sides and the tops of the cubes) may well be aligned with segments on the other. Subtle characteristics like these demand a high resolvability and low scattering in the Hough space. Figure 4-1b shows a 512×512 image of an office scene. It is more complicated than the image of the rubic cubes. Edge lines of various length and spatial distribution are observed. This normally represents a difficult case for standard HT methods. However, our results show that the HPCHT greatly improved the performance of the line detection in both examples.

Figure 4-2 shows the edge maps of the rubic cubes and the office scene respectively. A set of 3×3 Sobel-like edge masks were used to calculate the edge location and orientation. The resulting edge points were then filtered with a non-maximum suppression method [8] to obtain thin edges. It was observed that

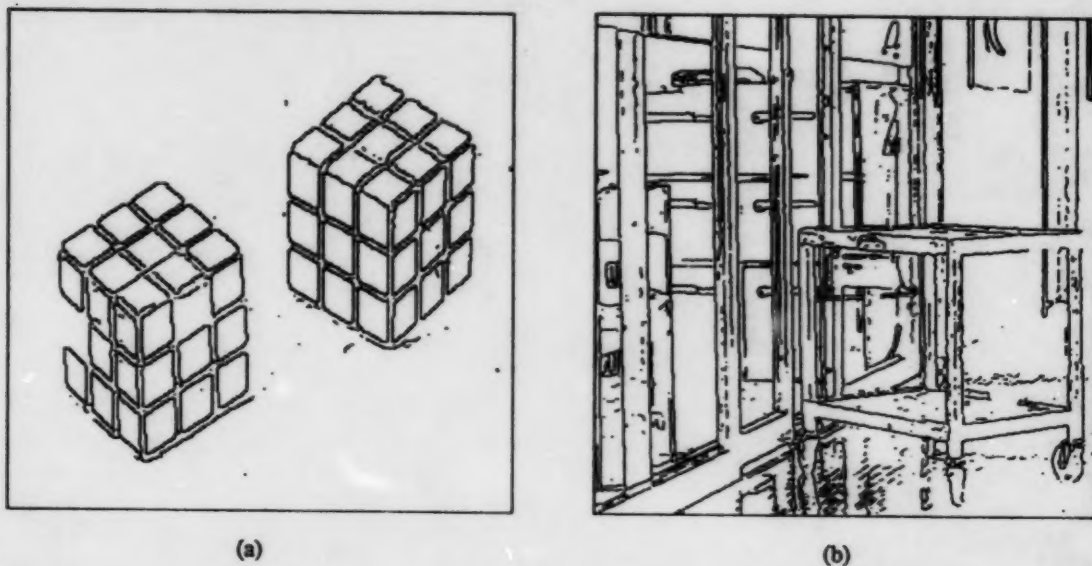


Figure 4-2:

Edge maps for the images in Figure 4-1. (a) The rubic cubes, $\sigma_{\Delta\theta} = 5^\circ$. (b) The office scene, $\sigma_{\Delta\theta} = 6^\circ$.

the θ estimates at different points on an edge line vary with a standard deviation of 5° for the cube image and with a slightly greater value at 6° for the office scene. These readings were taken as our initial $\sigma_{\Delta\theta_E}$ for the edge data.

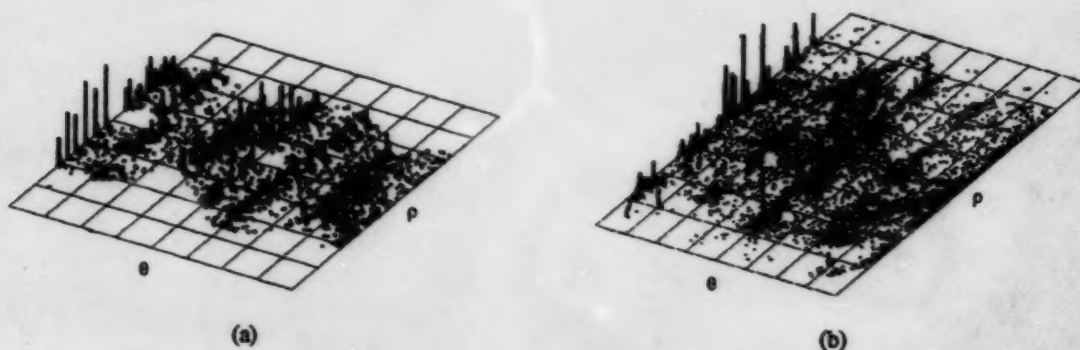


Figure 4-3:

The vote distribution from the standard HT on the two edge images. (a) The rubic cubes with a Hough space quantization of 1° and 3 pixels. (b) The office scene with a Hough space quantization of 1° and 3 pixels.

The standard HT methods easily fail in the presence of severe errors in the θ values. As indicated

by Eq. 4 in Section 2, errors in θ is manifested in ρ -scattering of the Hough voting. Figure 4-3 clearly shows that the errors in θ and ρ widely scatter the votes in the Hough space in both θ and ρ directions. Although some peaks are still observable, their positions are difficult to locate. In Figure 4-3b, the ρ -scattering is so severe that multiple peaks are practically fused together even though the Hough space is quantized at a fine scale of 1° and 3 pixels.

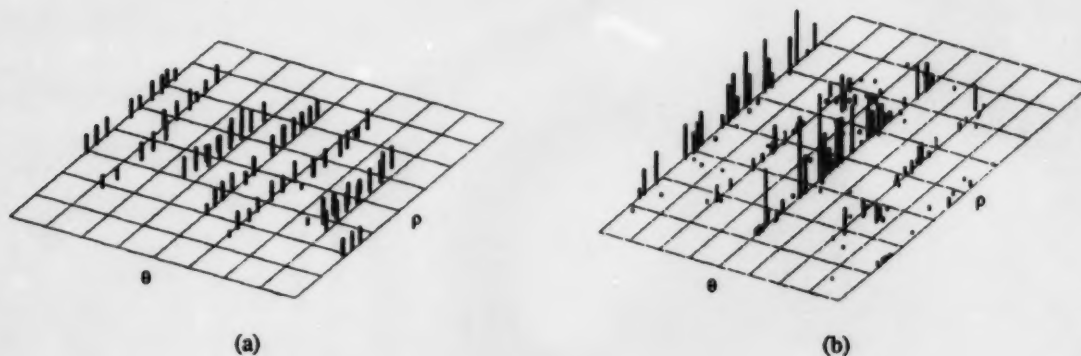


Figure 4-4:

The vote distribution from the HPCHT on the two edge images. (a) The rubic cubes with a Hough space quantization of 1° and 3 pixels. (b) The office scene with a Hough space quantization of 1° and 3 pixels.

Our method starts with small subimages where the ρ -scattering is reduced to within the tolerance allowed by the spatial separation of the image lines. As observed in Figure 4-2, the edge distribution of the cube image is sparse whereas the office scene has a much denser edge map. Parallel edges are more than 12 pixels away from each other in the former image but as close as 6 pixels in the latter. In our experiments, we chose a τ of 6 pixels for the cube image, and a τ of 3 pixels for the office scene. The HPCHT splits the image from top down the image hierarchy and then merges the segments extracted from bottom up. The Hough voting extracts the segments whose line parameters are evaluated by averaging the estimates from individual votes. The result is a smaller $\sigma_{\Delta\theta}$ as explained in Section 3.1. This accuracy improvement recursively enables an increasing precision in the estimates for the line parameters, which in turn keeps the vote scattering within the bound. Figure 3-6 has already demonstrated the increasing precision in the Hough space for three successive levels in the pyramid; and Figure 4-4 in this section shows the peaks at the top level. An accuracy of $\pm 1^\circ$ and ± 3 pixels is achieved on both test images. The ambiguity problem illustrated in Figure 4-3 vanishes in Figure 4-4. Peak detection is straightforward in the latter case.

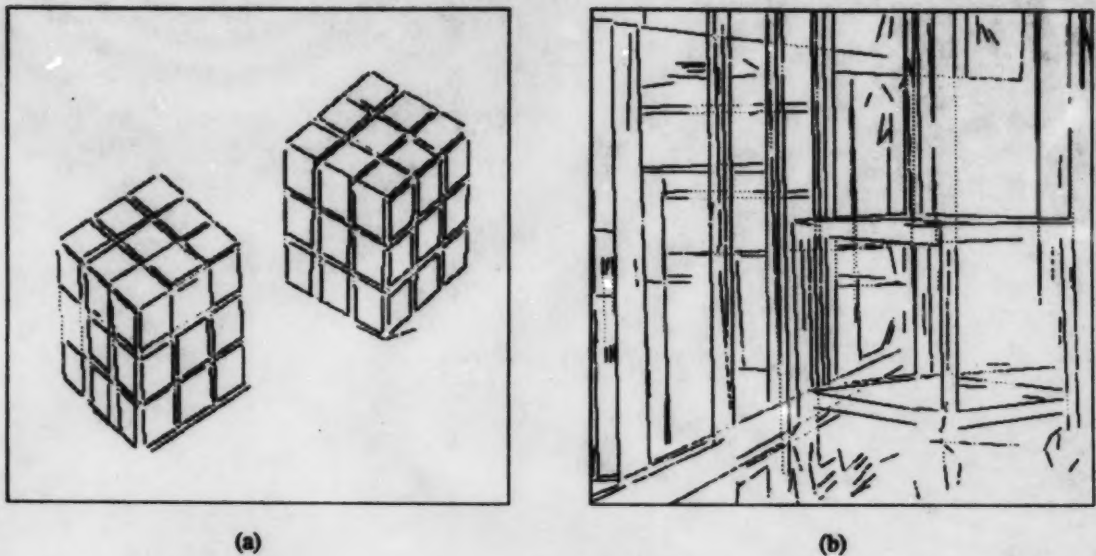


Figure 4-5:

The line images reconstructed from the lines extracted using the HPCHT method.
 (a) The rubic cubes. (b) The office scene.

We illustrate the accuracy of our line extraction algorithm with Figure 4-5. The extracted lines are depicted in the figures as lightly shaded segments on which the contributing edge points are plotted as dark pixels. The collinear relationship among the edge points is clearly shown. Comparing the result with the edge maps in Figure 4-2, we see that almost all the long edge lines have been extracted successfully by the HPCHT. The algorithm is robust since collinearity of partially occluded segments is recovered correctly.

5. Conclusion

In this paper, we pointed out that errors in the gradient direction are the major factor contributing to vote scattering in the Hough parameter space. This error is further aggravated when edge points are distant from the foot of normal of the underlying edge line. We showed how the scattering increases with larger images. These problems led us to the Hierarchical Peak Compaction Hough Transform (HPCHT), an algorithm which uses a low scattering HT procedure on small subimages initially, and then a recursive HT merging scheme on the extracted line segments in the subsequent stage. We were able to observe the

mathematical behaviour of the scattering and thus came up with a bound on vote scattering which guides the recursive subdivision of the image in the first stage, and the adaptive quantization of the parameter space in the second stage of the algorithm. As a result, low ρ -scattering and high θ -precision are achieved. Experimental results for the recursive HPCHT algorithm were shown to confirm our analysis. Tests performed on real data generated sound results. Almost all lines in the test images were correctly extracted. The method was demonstrated to be effective and accurate with interrupted lines being properly connected, and separate but nearly aligned segments being correctly isolated at the same time.

The hierarchical approach to HT has also been explored by other researchers. In the Adaptive HT of Illingworth and Kittler [7], they implemented a multiresolution peak finding search over the Hough space. The Fast HT of Li et al [9] used an iterative coarse to fine exploration based on a quadtree decomposition of the parameter space. The main thrust of their work was on the issue of efficiency in computation. Both cpu time and storage requirements were improved by taking advantage of the fact that the accumulator array is mostly sparse. Our work, however, was initiated by the fuzzy peaks in the Hough space. Postprocessing using peak sharpening techniques [4] may not work because the scattering effect is so severe for large images. Votes from irrelevant edge points may overlap and mistakenly be assigned to the same peak.

Recent work of Princen, Illingworth and Kittler [10] employed a bottom-up integration approach in their Hierarchical HT to extract image lines in the global space by grouping short segments detected from local neighbourhoods. Their results are related to ours, except that the theme of their work is to use a hierarchical structure to achieve recursive collinear grouping for efficiency and to enforce a proximity constraint to filter out irrelevant groupings due to accidental alignment. While the method is efficient and able to avoid random line segments, it cannot, however, deal with the problems of inaccurate edge data. We have shown in this paper that errors in the edge data affect the extent of vote scattering which determines directly the sampling rate and also the peak detection strategy for the vote clusters in the Hough space.

In this paper, we have addressed the problem of accuracy of the HT line extraction. Our contribution is a better understanding of the scattering effect of vote clusters, and we presented a peak compaction procedure for HT line extraction which is effective and accurate. The algorithm employs a hierarchical approach which also inherits the merits of proximity constraint and the efficiency of local grouping. The hierarchical structure of the recursive line formation also facilitates independent execution of the Hough procedures on individual local subimages. This lends to a potential parallel implementation

on pyramid computers or other suitable architectures.

The method presented in this paper can be extended to image pattern recognition on geometrical structures other than lines. Hough methods on circular patterns in an image face the same problems of vote scattering when the edge data are corrupted. On large circles, the edge points tend to form a vague cluster of votes in the Hough space rather than voting precisely to a single point at the circle center. A hierarchical approach of resolution reduction based on alternating pixel selection would be able to reduce the circles to coarse resolution in small subimages and thus limit the vote scattering. In the same way, recursive merging from coarse to fine scales would merge the clusters which vote to the same circle center to a fine resolution while maintaining the vote scattering at low magnitude. An immediate extension of this work will be testing the ideas behind the HPCHT on circle detection, while future research would extend the idea further to the Generalized Hough Transform (GHT) on arbitrary image patterns. Similar problems of vote scattering are also anticipated in the GHT, and the hierarchical approach is expected to yield better results.

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References

- [1] P. J. Burt.
The Pyramid as a Structure for Efficient Computation.
In A. Rosenfeld (editor), *Multiresolution Image Processing and Analysis*, pages 6-35. Springer-Verlag, 1984.

- [2] J. Canny.
A Computational Approach to Edge Detection.
IEEE Trans. PAMI (8):679-698, 1986.
- [3] M. Cohen and G. T. Toussaint.
On the Detection of Structures in Noisy Pictures.
Pattern Recognition (9):95-98, 1977.
- [4] G. Gerig and F. Klein.
Fast Contour Identification through Efficient Hough Transform and Simplified Interpretation Strategy.
In *Proc. 8th International Joint Conference on Pattern Recognition*, pages 498-500. Paris, 1986.
- [5] F. O'Gorman and M. B. Clowes.
Finding Picture Edges through Collinearity of Feature Points.
IEEE Trans. Comput. (C-25):449-456, 1976.
- [6] P. V. C. Hough.
A Method and Means for Recognizing Complex Patterns.
U.S. Patent 3,0690,654, 1962.
- [7] J. Illingworth and J. Kittler.
The Adaptive Hough Transform.
IEEE Trans. on PAMI (9):690-697, 1987.
- [8] Z. N. Li.
Dynamic Programming in Hough Space for Line Matching in Stereopsis.
In *SPIE Symposium on Advances in Intelligent Robotics Systems '89, SPIE Vol. 1195 (Mobile Robots IV)*, pages 209-220. Philadelphia, 1989.
- [9] H. Li, M. A. Lavin, R. J. McMaster.
Fast Hough Transform: a Hierarchical Approach.
CVGIP (36):139-161, 1986.
- [10] J. Princen, J. Illingworth and J. Kittler.
A Hierarchical Approach to Line Extraction.
In *Proc. IEEE CVPR Conf.*, pages 92-97. San Diego, 1989.
- [11] A. Rosenfeld, J. Ornelas and Y. Hung.
Hough Transform Algorithms for Mesh-Connected SIMD Parallel Processors.
CVGIP (41):293-305, 1988.
- [12] A. Rosenfeld.
Picture Processing by Computer.
Academic Press, New York, 1969.
- [13] T. M. Van Veen and F. C. A. Groen.
Discretization Errors in the Hough Transform.
Pattern Recognition (14):137-145, 1981.

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